Modelling and Control Design
for a Hydraulic Forestry Crane

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Abstract

In the industry of forestry cranes, the design of automated machines that improve productivity has been a challenge for many researchers and engineers in the field of Control Systems and Robotics. An appropriate control design requires a detailed knowledge of the physical phenomena involved in the real system. At this point it is needed to perform simulation tests more rapid than experimental ones. In order to run those simulations, a model of the real machine is required. In this project the MATLAB/Simulink package for both modelling and simulation has been used. Basic mathematical models of components involved in the system as well as the final block diagrams are presented in this report.

In heavy-duty machinery, friction may reduce the performance significantly and negative side effects such as tracking errors, large settling times or limit cycles are usually introduced. In todays high-performance motion systems such effects have to be taken into account in the control design in order to gain accuracy. For this reason certain types of tests must be performed to understand and interpret the friction phenomena hidden in the system. The identification of friction parameters based on experiments is explained in this report and a corresponding model for the system is presented.

The derived control system is based on an empirical tuned PID controller with an additional feedforward term for friction compensation. Generally, friction models require the velocity as input. Due to the fact that only positions are measured, an elaborate estimation algorithm is proposed. As result of this report, two examples of an autonomous motion are presented and an outlook on further improvements is given. A comparison of the derived simulation model with the behaviour of the real machine is also shown.
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1 Introduction

Cranes have to perform several motions that are commonly used to lift heavy loads. Hydraulic motors are especially suited to fulfil this main demand. In addition, there are much more functions that can be provided applying hydraulic components. Usually, forestry machines like harvester and forwarder are using hydraulic power for two main purposes:

- the mechanical transmission, i.e. the propulsion, and
- the crane hydraulics itself.

A diesel engine supplies power to a hydrostatic transmission unit that distributes it finally via pumps according to the demands of the operator. Since the power is shared, it could happen that the engine stalls if the maximum is exceeded. This should be avoided in order to prevent from instantaneous loss of control. The hydraulic transmission will not be discussed here. The crane hydraulics can be grouped into the following parts:

- the pump
- the control valve unit and
- the actuators, i.e. cylinders and rotational motors.

Each actuator enables one function at the crane generating appropriate forces at the mechanical devices. In Figure 1 the crane is shown in 2-dimensional space. The drawing is made according to the existing experimental set-up in the crane laboratory at Umeå University. There are sensors for the angle of the first link $\theta_2$ and the angle of the second link $\theta_3$ as well as for the telescope position $d$.

![Figure 1: Crane mechanics with the hydraulic actuators.](image)

In order to analyse the crane behaviour and to perform an appropriate control design a mathematical model has to be derived. Hydraulic components and their combination with the mechanical part of the crane will be presented in chapter 2. Eventually, the analysis and consequently the model will be reduced to the dynamics of the first link (chapter 3).
Naturally, friction shows up when the crane is moving. Due to the facts that the crane components are mounted together and that their several friction models are not known, an appropriate lumped model has to be obtained by identification. How this can be done is described in chapter 4.

A detailed parametrisation of the first-link-model will be given in chapter 5. Preliminary steps in control design including friction compensation are presented in chapter 6. An outlook on further steps is given as well. In chapter 7, two examples of an autonomous motion are presented as experimental results. A comparison of the derived simulation model with the behaviour of the real machine will be also shown. Finally, this report ends with essential conclusions.
2 Modelling of the hydraulic components

2.1 Pump

A pump is the generator of the fluid power. To keep a constant pressure a load sensing system is used. It is a type of pump control employed in open circuits [8]. The pump works in a variable way with a variable displacement that controls the angle that the pump uses to give a desired pressure. The load sensing is a hydraulic feedback method by which an orifice (commonly a directional control valve) is used to provide the sensed pressure as reference to the pump. The displacement varies according to the error desired and constant pressure.

2.2 Control valve unit – four-way valve

Usually, a control valve unit consists of several 4/3–valves (i.e. 4 ways and 3 positions) regulating the volumetric flow to the connected actuators. These valves are solenoid operated, so that the port opening can be achieved by a current flow through the coils. A magnetic field provides the electromotive force to move the especially shaped valve spool. This motion is opposed by a centering spring, so that the flow to all ports is blocked in the de-energised mode. Furthermore, the shifting behaviour is damped in that way. A changing spool position results in varying orifice areas of the appropriate ports in order to adjust the volumetric flow. Such a proportional directional control valve is illustrated in Figure 2 as block symbol. With two solenoid assemblies S1 and S2 it can provide control of direction as well as of flow. The ports are represented by the pump P, the return tank T, the actuator inlet chamber A and the actuator outlet chamber B.

Figure 2: Block symbol of a proportional 4/3–valve [7].

However, a 4/3–valve is a complex device that reveals a high-order non-linear response. An accurate mathematical model requires a large number of non-measurable internal parameters, such as orifice sizes, spring rates, spool geometry etc. The manufacturers tune those parameters to generate the specific valve behaviour, which can be represented as frequency response, step response, change of flow in respect of supplied pressure or spool position etc.

The non-linearity of the 4/3–valve is due to the physical construction made to perform electro-hydraulic tasks. To derive a mathematical model of such a complex system, it is often convenient to ignore any inherent non-linearities and employ a small perturbation analysis. Hence, the real behaviour and performance can be approximated by a linear model. Assuming that the volume of the valve chambers are negligible compared to the overall volume of the system and that the volumetric flow can be included in the dynamic model of the actuators due to compressibility effects, the flows through the valve can be calculated by algebraic equations [11]. Furthermore, the instantaneous flow can be assumed to be equal to steady state conditions at a given fluid pressure and constant spool position; i.e. the transitional flow at each instant of spool movement is neglected. As a result of the assumptions, the valve model can be decoupled into two independent parts:
• the dynamics of the valve spool (dynamic model) and
• the flow characteristics at a given spool position and fluid pressure (static model).

2.2.1 Static model of a 4/3-valve

A flow \( q \) through a restriction or orifice is generally turbulent and given by

\[
q = C_d A \sqrt{\frac{2}{\rho} \Delta p}
\]  

(1)

where \( A \) represents the cross section of the orifice, \( \Delta p \) the pressure drop and \( \rho \) the density of the fluid [3]. In practice there is some loss of energy over the orifice that shows up in the constant discharge coefficient \( C_d \). The flow is basically affected by the pressure drop and the opening of the orifice.

Mostly, the current values of the orifice areas are not known or measurable; also \( \rho \) might be not stated. For this reason a more general relation has to be used in order to represent the flow in respect of the current valve spool position. According to [4],[5] and [10], a convenient equation can be derived by introducing a flow coefficient

\[
K(x_v) = \frac{Q_N}{\sqrt{\Delta P_N}} A_R(x_{v,R}) \approx C_d A \sqrt{\frac{2}{\rho}}
\]  

(2)

where the parameters nominal volumetric flow \( Q_N \) and nominal pressure difference \( \Delta P_N \) are static valve characteristics provided in data sheets. Normally, the flow is given as a hyperbolic function of the spool position to describe the opening (see Figure 3). A so-called pseudo-section \( A_R(x_{v,R}) \) can be applied with normalised values of the flow in respect of \( x_{v,R} = [-1, 1] \) as normalised position of the valve spool.

![Figure 3: Typical curves showing flow as a function of spool stroke [13].](image)

In the origin of the opening process the restriction is quite narrow, so that the volumetric flow tends to zero. This means that it has to be assumed as laminar [3]. Hence, the flow equation (1) changes to

\[
q = \begin{cases} 
K(x_v) \Delta p & |\Delta p| < p_{tr} \\
\frac{K(x_v) \Delta p}{2 \sqrt{p_{tr}}} \left(3 - \frac{|\Delta p|}{p_{tr}}\right) & |\Delta p| \geq p_{tr}
\end{cases}
\]  

(3)
where a smooth transition between the laminar and turbulent regimes has been achieved. Negative flows for $\Delta p < 0$ are also taken into account. The transition pressure

$$p_{tr} = \frac{9 \text{Re}_{tr}^2 \rho v^2}{8 C_d^2 D^2}$$

(4)

corresponds to a given threshold $\text{Re}_{tr}$ for the Reynold number. $D$ represents the diameter of the restriction and $\nu$ the kinematic viscosity of the fluid. For high Reynold numbers ($\text{Re} > 1000$) the flow through a restriction will be turbulent [3]. Generally, the transition pressure $p_{tr}$ is in the range of 2 to 4 bar.

The general functionality of a matched and symmetric 4/3-valve is shown in Figure 4 and Figure 5. Here, the (pseudo-) cross sections of the orifices $a$, $b$, $c$ and $d$ are equal two-by-two [3] and the port openings are controlled by the spool position $x_v$, i.e.

$$K_a(x_v) = K_d(x_v) = K_b(-x_v) = K_c(-x_v).$$

(5)

The appropriate pressures and flows of the four ports are indicated by $P$ for pump, $T$ for tank, $A$ for the actuator input and $B$ for the actuator output.

Figure 4: Orifice flows through a four-way valve [3].

Figure 5: Working principle of a matched and symmetric 4/3-valve [3].
According to Figure 4 and Figure 5, the port flows are formed by the orifice flows through the equations

\[ q_P = q_a(x_v, p_P, p_A) + q_b(x_v, p_P, p_B) \]
\[ q_T = q_c(x_v, p_A, p_T) + q_d(x_v, p_B, p_T) \]
\[ q_A = q_a(x_v, p_P, p_A) - q_c(x_v, p_A, p_T) \]
\[ q_B = q_d(x_v, p_B, p_T) - q_b(x_v, p_P, p_B). \] (6)

Considering the previous equations (2) till (6), a static model of the 4/3-valve has been achieved (see Figure 6) with the inputs spool position \( x_v \) and the pressures \( p_P, p_T, p_A, p_B \). As outputs the flows \( q_A \) and \( q_B \) have to be connected to the actuator.

\[ \begin{align*}
\chi p_P [{\text{Pa}}] & \quad q_P [{\text{m^3/s}}] \\
\chi p_T [{\text{Pa}}] & \quad q_T [{\text{m^3/s}}] \\
\chi p_A [{\text{Pa}}] & \quad q_A [{\text{m^3/s}}] \\
\chi p_B [{\text{Pa}}] & \quad q_B [{\text{m^3/s}}] \\
\chi x_v [{\text{m}}] & \quad q_B [{\text{m^3/s}}] 
\end{align*} \]

Figure 6: Model block of the static 4/3-valve

### 2.2.2 Spool dynamics

The valve spool dynamics is very important because the spool position \( x_v \) is the control input to the derived static valve model. A dynamic analysis leads to a more complex model in order to represent the real valve performance.

Generally, valves are controlled by electronic devices so that the transformation of electrical signals into spool position is focused on. Commonly, a PWM driver is used to convert an input current given by the electronic controller device into a modulated current that energize the internal solenoids drivers. The respective magnetic force will move the spool to a desired position \( x_p \), i.e. the speed of hydraulic actuators can be proportionally adjusted. According to [15], the solenoid force can be simplified as a linear function of the input current \( i \) to the PWM driver. Hence, the spool dynamics is given by

\[ F = Bi = kx_p + cx_p + mx_p, \] (7)

where \( B \) represents the total coefficient including PWM, coil and solenoid, \( k \) the spool spring stiffness, \( c \) the proportional damping factor and \( m \) the total mass of the spool. For this reason, it is usually only necessary to accurately model valve response through a relatively low range of frequencies, and the servo-valve dynamics may be approximated by a second order transfer function without serious loss of accuracy. As mentioned before, the manufacturers specify this dynamic behaviour with curves for frequency response curves and sometimes the step response is also given.

A typical frequency response of a proportional valve is depicted in Figure 7. The characteristics such as natural frequency response \( \omega_v \), the damping \( \delta_v \) and the transfer function
Figure 7: Typical frequency response of a proportional valve [4].

\[ G_v(s) \] are taken from the graph using the next formulas [16]:

\[
G_v(s) = \frac{1}{\tau_2^2 s^2 + \tau_1 s + 1}
\]

with \( \omega_v = \frac{1}{\tau_2} \), \( \tau_1 = \frac{\tau_2}{|G|_{\omega_{90^\circ}}} \) and \( \delta_v = \frac{\tau_1}{2 \tau_2} \)  \hspace{1cm} (8)

Another option to represent and to find this transfer function is the use of the MATLAB System Identification Toolbox. It is possible to determine the direct transfer function having the frequency response of the system by using the command ident implemented in MATLAB. This opens the possibility to try higher order transfer functions.

The model for the spool dynamics contains two non-linearities: limitations in velocity \( L_v \) and acceleration \( L_a \) [4]. Besides the phase lag has to be adjusted with a delay \( \Delta t \) to match the phase of the response. The block diagram for the spool dynamics is shown in Figure 8.

Figure 8: Block diagram for the spool dynamics [4].

Finally, a model of the 4/3-valve with the dynamic and static model subsystems can be built (see Figure 9).

2.3 Cylinder

Hydraulic motors are much more powerful in comparison to electrical ones of the same dimensions. In principle, linear and rotational hydraulic motors work in the same way. The pressures of the inlet an outlet chambers are used to create a force or torque respectively.

Linear hydraulic motors are represented by cylinders. Single-rod cylinders (see Figure 10) are often used in cranes because of the higher performance in comparison to symmetric
ones. The generated motor force on the piston will be:

\[ F_p = A_A p_A - A_B p_B \]  

(9)

where \( A_A = A_p \) and \( A_B = A_p - A_r \) with the cross sections of the piston \( A_p \) and the rod \( A_r \). The volumes for the inlet and outlet chambers are defined as

\[ V_A = V_{A0} + A_A x_p \quad \text{and} \quad V_B = V_{B0} + A_B (x_{p,\text{max}} - x_p). \]  

(10)

The maximum distance where the piston can move to is given by the stroke \( x_{p,\text{max}} \). The dead volumes \( V_{A0} \) and \( V_{B0} \) are formed by the extension parts of the cylinder that are used to connect the hydraulic pipes.

According to [3], the mass balance for a volume \( V \)

\[ \frac{V}{\beta} \dot{p} + \dot{V} = q_{\text{in}} - q_{\text{out}}, \]  

(11)

where \( \beta \) represent the bulk modulus. This equation leads to coupled differential equations for the two chamber pressures

\[ \dot{p}_A = \frac{\beta}{V_A} [-C_{im}(p_A - p_B) - C_{em} p_A - A_A \dot{x}_p + q_A] \]

\[ \dot{p}_B = \frac{\beta}{V_B} [-C_{im}(p_B - p_A) - C_{em} p_B + A_B \dot{x}_p - q_B]. \]  

(12)

Finally, a model block of the cylinder can be obtained (see Figure 11). Here, the chamber flows \( q_A \) and \( q_B \) are taken from the control valve as inputs. According to the mechanical system where the cylinder applies the generated motor force \( F_p \), the resulting dynamics will give the position \( x_p \) and the velocity \( \dot{x}_p \) as further inputs. The outputs chamber pressure \( p_A \) and \( p_B \) have to be linked back to the control valve.
3 Combining the crane mechanics with the hydraulic components

3.1 Introduction

Cranes perform their tasks with the help of hydraulic actuators that are connected to the several links and bearings. In Figure 12 the crane is shown in 2-dimensional space. The drawing is made according to the experimental setup with lines representing the several beams. There are sensors for the angle of the first link \( \theta_2 \) and the angle of the second link \( \theta_3 \) as well as for the telescope position \( d \). In order to derive a mathematical model, the crane mechanics has to be combined with the hydraulic components. In the following, the analysis will be limited to the first link.

3.2 Ideal model for the first link

Generally, the modelling process is based on physical laws. The resulting model will be ideal because the friction that occurs in the real world cannot be determined from scratch. A friction model will be obtained in the following chapter based on identification test.

In order to combine the mechanical part of the first link with the appropriate hydraulic cylinder, the geometric relations have to be examined. If the piston position of cylinder is adjusted, there will be a corresponding change of the angle \( \theta_2 \) and thus the first link will move up and down respectively. In Figure 13 the geometric connections of the first link components are shown. Eventually, a triangle is formed through the cylinder with an angle \( \theta_2 \) representing the position of the link and a varying side \( x(x_p) \) according to the piston position. The adjacent sides \( j_1 \) and \( j_2 \) as well as the needed angles \( \varphi_3 \) and \( \varphi_4 \) are fixed and given by the crane dimensions from out of Figure 12. Applying simple trigonometry and the law of cosine [1], the following equations can be obtained:

\[
\begin{align*}
  j_1 &= \sqrt{r_1^2 + d_1^2} \\
  j_2 &= \sqrt{(r_3 - r_4)^2 + (d_2 - d_4)^2} \\
  \varphi_4 &= \arctan \left( \frac{d_1}{r_1} \right) \\
  \varphi_3 &= \arcsin \left( \frac{d_2 - d_4}{j_2} \right) \\
  \varphi_2(x_p) &= \frac{\pi}{2} - \varphi_4 + \theta_2(x_p) + \varphi_3 \\
  x(x_p)^2 &= j_1^2 + j_2^2 - 2j_1j_2 \cos (\varphi_2(x_p)).
\end{align*}
\]
Hence, the angle \( \theta_2(x_p) \) and thus the position of the link can be computed in respect of a given piston position:

\[
\begin{align*}
\varphi_2(x_p) &= \arccos \left( \frac{j_1^2 + j_2^2 - x(x_p)^2}{2 j_1 j_2} \right) \\
\theta_2(x_p) &= \varphi_2(x_p) - \frac{\pi}{2} + \varphi_4 - \varphi_3.
\end{align*}
\]

Finally, the force on the link has to be determined that generates a moment around the joint. It will be less than the cylinder force \( F_{cyl}(p_A, p_B) \) due to the angle \( \gamma(x_p) \). An increasing piston position \( x_p \) will enlarge this angle. The effective force component on the link \( F(p_A, p_B, x_p) \) can be computed as follows:

\[
\begin{align*}
\varphi_1(x_p) &= \arccos \left( \frac{j_2^2 + x(x_p)^2 - j_1^2}{2 j_2 x(x_p)} \right) \\
\gamma(x_p) &= \frac{\pi}{2} - \varphi_1(x_p) - \varphi_3 \\
F(p_A, p_B, x_p) &= F_{cyl}(p_A, p_B) \cos(\gamma(x_p)).
\end{align*}
\]

With the force acting on the first link the equation of motion can be formulated by

\[
\dot{\theta}_2 = \frac{1}{J} (\tau - m_t gd_m).
\]

The inertia of the first link is denoted by \( J \) and is estimated under the assumption of having a total mass \( m_t \) rotating around the joining point at the distance \( d_m \). It follows

\[
J = m_t d_m^2.
\]
with \( d_m = r_3 - r_4 \) as the distance to the acting force. The torque generated by the cylinder will be

\[
\tau = F(p_A, p_B, x_p) d_m. \tag{18}
\]

Finally, the resulting block diagram of the ideal model for the first link can be built as shown in Figure 14. As already mentioned, the piston position \( x_p \) and its velocity \( \dot{x}_p \) have to be fed back from the mechanical block to the cylinder block.

---

Figure 13: Geometry of the first link.

Figure 14: Block diagram of the ideal model for the first link.
4 Modelling of friction by identification

4.1 Introduction to friction models

4.1.1 Definition of friction

Friction occurs as tangential reaction force between two surfaces in contact. Basically, it shows up in all kinds of mechanical systems like in bearings, transmissions, brakes as well as in hydraulic and pneumatic cylinders, valves etc.. The physical phenomena that finally result in friction forces are detailed discussed in [2], [3] and [12]. Eventually, friction shows highly non-linear attributes depending on numerous constraints such as contact geometry, properties of the surface materials, displacement and relative velocity of the bodies and presence of lubrication.

In the presented modelling process for the crane, friction was not taken into account at all. There is simply no way to derive an appropriate friction model only from physical laws. This fact automatically leads to an empirical approach in order to construct a model by reproducing effects observed in experiments.

4.1.2 Static friction models

Commonly, static friction models represent a straightforward solution in empirical modelling. Here, the friction force is mapped as a function of relative velocity. For the individual phenomena several models are known and combined respectively (see Figure 15). The functions do not have to be necessarily symmetric to the point of origin; the magnitude of the friction force may vary for different directions of the velocity. In the following, the basic static friction models will be briefly examined for a better understanding of the phenomena behind.

**Figure 15:** Selection of static friction models [12]: a) Coulomb friction, b) Coulomb plus viscous friction, c) Stiction plus Coulomb and viscous friction and d) Stribeck effect.

**Coulomb friction:** The Coulomb friction is the classical idea where the friction force opposes the motion and its magnitude is independent of velocity and contact area [12]. Consequently, the Coulomb model is given by

\[ F_f = F_C \text{sign}(v), \quad v \neq 0, \]

\[ (19) \]
where the Coulomb force $F_C$ is proportional to the normal load, i.e. $F_C = \mu F_N$.

**Static friction – stiction:** In the case of stiction where the relative velocity is zero, static friction is present. By increasing an external force, the system starts slipping (break-away) at a certain magnitude that represents the stiction force $F_S$. However, static friction, as opposed to dynamic friction, describes the friction force at rest and counteracts external forces below a certain level and thus keeps an object from moving [12]. It is modelled using the applied external force $F_e$ at zero velocity:

$$F_f = \begin{cases} F_e & \text{if } v = 0 \text{ and } |F_e| < F_S \\ F_S \text{ sign}(F_e) & \text{if } v = 0 \text{ and } |F_e| \geq F_S \end{cases}$$

(20)

In some cases the friction force might be larger in magnitude for zero velocity than for non-zero, i.e. that the stiction force $F_S$ is larger than the Coulomb force $F_C$ then [3].

**Viscous friction:** Viscous friction is caused by the viscosity of lubricants. In fluid lubricated contacts the friction force also depends on the magnitude of the velocity due to hydrodynamic effects [3]. Usually, viscous friction is described by a linear model as follows:

$$F_f = F_V v,$$

(21)

where $F_V$ represents the constant of proportionality depending on lubricant viscosity, loading and contact geometry [3].

**Strubeck effect:** According to [3], the Strubeck effect has its background in partial fluid lubrication in the transition phase from sticking to sliding. In this case some of the load is carried by the lubrication and some by elastic and plastic deformation of the microscopical asperities on the surface. Hence, the magnitude of the friction force does not decrease discontinuously as it is indicated by the previous models, the velocity dependence is rather continuous. The Strubeck effect is usually given by

$$F_f = \left[ F_C + (F_S - F_C)e^{(v/v_s)^2} \right] \text{ sign}(v), v \neq 0$$

(22)

with the stiction force $F_S$, the Coulomb force $F_C$ and the characteristic Strubeck velocity $v_s$. The resulting curve is assigned to fit to experimental data and is not based on the physics behind this phenomenon.

**Constraints in static models:** Static models cannot cover dynamic effects such as depicted in Figure 16. There is a phenomenon called pre-sliding displacement that makes motion possible, even when a system is stuck in static friction. This effect can be explained

Figure 16: Friction phenomena that are not covered by static friction models [12].
by the spring-like behaviour before actual sliding occurs [3]. Releasing the applied force may result in a permanent displacement. Furthermore, the break-away force varies according to the rate of change of the external force applied to the system. A higher force rate gives a smaller break-away force. A very important effect shows up in unidirectional experiments, where the magnitude of the friction force is lower for decreasing than for increasing velocities. This results in a hysteresis loop (frictional lag). All the mentioned phenomena may cause wide mismatches in simulation and control applications while applying static models.

4.1.3 Dynamic friction models

Obviously, there are two problems connected to the use of static friction models:

- The models rely on switching at zero velocity and thus depend on its detection.
- They cannot cover all observed dynamic effects such as pre-sliding displacement, varying break-away force and hysteresis.

Dynamic friction models attempt to solve these problems. As major difference to the empirical motivated static models, the physics behind the several phenomena are tried to capture. The Dahl model represents a first step in development. For small displacements, the friction is determined by elastic deformation in the pre-sliding phase. The microscopical asperities on the surface are assumed as linear springs with appropriate stiffness [3]. The model is a generalisation of ordinary Coulomb friction where the friction force is only a function of the displacement and the sign of the velocity [12]. With the LuGre model a more elaborate design was proposed. It is based on the idea of a dynamic bristle model for the asperity junctions of two bodies in contact. The friction force is generated by a dynamic equation describing the average deflection of the bristles [3]. As advantage in comparison to the Dahl model, stiction and the Stribeck effect are included in the LuGre model. Moreover, hysteresis can be represented. Dynamic friction models are extensively discussed in [2] and [12].

However, dynamic models are also based on experimental identification. Characteristic friction values like the Coulomb force $F_C$ (sufficient for the Dahl model), the stiction force $F_S$, the constant of proportionality $F_V$ for viscous friction and the Stribeck velocity $v_s$ have to be known for an accurate model. Furthermore, the properties of the bristles or springs respectively have to be determined.

4.2 Identification

4.2.1 Theoretical approach

A corresponding friction model for each actuator of the crane, such as the first and second link, the telescope, the gripper etc., can be derived by experimental identification. According to Newton’s laws of motion, the sum of all forces acting on an object has to be zero. Considering the presence of friction at a control actuated device, it follows:

\[
\begin{align*}
\text{for translation:} & \quad m\ddot{x} = u - F \\
\text{for rotation:} & \quad J\ddot{\theta} = u - F_\tau
\end{align*}
\]

(23)

In the case of translational motion, $m$ represents the mass of the object and $\ddot{x}$ its acceleration driven by an actuated control input $u$ as external force minus the opposed friction force $F$. Respectively, the objects moment of inertia $J$ and its angular acceleration $\ddot{\theta}$ driven by an actuated control input $u$ as external torque minus the opposed friction torque $F_\tau$ show up in the rotational equation of motion. Basically, the equations (23) reveal an
opportunity how to identify friction even when it is not directly seen while running the crane. Observations let assume that a constant control input is proportional to a resulting steady state velocity. In fact this is a reasonable evidence for the presence of friction in the system. Eventually, the identification of friction can be done by experiments with constant velocity. If the acceleration is zero, the equations (23) are simplified as follows:

\[
\begin{align*}
\text{for translation:} & \quad 0 = u - F \\
\text{for rotation:} & \quad 0 = u - F_r
\end{align*}
\]

(24)

Under static conditions the friction force or torque respectively is equal to a given external input. Consequently, a static friction map can be constructed by increasing the control input step by step in order to achieve a range of constant velocities beginning from zero. It is assumed that the resulting map gives approximately adequate values under dynamic conditions, i.e. during transient phases.

4.2.2 Obtaining a lumped pseudo-friction model

As main advantage of the previously described approach, the friction in all the several components of each actuator of the crane can be identified by once. This basically means that the friction in the bearings, in the hydraulic actuators, in the valves etc. are lumped in one model covered by the equations (24).

In order to achieve a static map including friction forces or moments respectively, the certain actuated control input \( u \) is required. Indeed, only the given current \( i \) as control input to the valve is exactly known. The resulting force or moment at the corresponding actuator of the crane is not measured and cannot be determined. Nevertheless, it is sufficient to exploit the control current \( i \) for friction purposes. Introducing a variable \( k \) that describes the unknown part of the actuated control input \( u \), it follows

\[ u = i k. \] (25)

Note that the dynamics from the given control current to the finally effected motion is hidden in this equation, i.e. dynamical effects due to the valve and the hydraulic actuator as well as occurring delays are not seen. Eventually, the unknown variable \( k \) will also show up in the friction term. Hence, the static equations (24) can be generally rewritten as

\[ 0 = (i - i_f)k \] (26)

with \( i_f \) as pseudo-friction expressed as current. For control purposes, the factor \( k \) itself is not very important; but it is a different one for translational and rotational motions and may depend on time. In conclusion, the friction can be identified by the given control current \( i \), even when \( k \) is unknown and thus the physical meaning of friction is not captured. For simulations and control applications, a convenient pseudo-friction model has to be constructed applying equation (26). The benefit, directly seen from this method, is that the control variable and the friction have the same physical expression. This can be used advantageously for friction compensation.

In the following, the identification of the lumped pseudo-friction model will be shown on the basis of the first link. The experiments have to be performed in open-loop mode in order to fulfil equation (26). Applying a square wave with a mean value equal to zero as input to the control valve seems to be reasonable, because the actuator works bidirectional (see Figure 17). At this, the magnitude of the current \( i \) has to be stepwise increased capture by capture in order to produce a range of constant velocities beginning from zero. The period of the square wave must be large enough, so that transitional effects can decay and
Figure 17: Friction identification via square-wave input in open-loop (1st link).

thus the acceleration remains zero for a while. Capturing the position, supplied through the corresponding sensor, the velocity and acceleration can be determined by filtering. Here, the filtfilt command from the Signal Processing Toolbox of MATLAB has been used in order to obtain precisely zero-phase distortion for the captured sequences. A simple high-pass filter with cutting frequency at 3 rad/s gives the velocity and acceleration respectively. In Figure 18 the analysis for each capture is demonstrated exemplified. All the values where the acceleration is equal to zero are used to evaluate the friction in order to get as much data points as possible. Eventually, more than 50 captures with thousands of points each have been used. Finally, a given control current $i$ results in a certain value for the pseudo-friction $i_f$ at a constant velocity.

In Figure 19 the static friction map for the first link is shown as outcome of the analysis. As it can be seen, the curve has been sufficiently approximated by an asymmetric Coulomb plus viscous friction model as follows:

$$i_f = \begin{cases} \alpha_2^+ \omega + \alpha_0^+ & \text{if } \omega < 0 \\ \alpha_2^- \omega + \alpha_0^- & \text{if } \omega > 0 \end{cases}$$

(27)

Here the angular velocity is denoted by $\omega$. The offset due to Coulomb friction is represented by $\alpha_0$, the slope due to viscous friction by $\alpha_2$. The plus and minus indicate the (positive) upper half plane of the static map and the (negative) lower half plane respectively. Note that a negative input current results in a positive velocity and thus the map seems to be mirrored to the ordinate. Furthermore, a higher input current is needed to lift the link in comparison to the lowering process. This might be due to the gravity or certain cylinder properties given by the manufacturer.

The identification procedure as demonstrated above has been repeated for the second link and finally the telescope using the already installed position sensors. The out-coming static friction maps are shown in Figure 20 and Figure 21 respectively.

In unidirectional experiments with increasing and decreasing velocity, the presence of hysteresis (frictional lag) could have also been observed. This effect is not captured by the static maps shown above.
First link – Sample 0268

Figure 18: Capture analysis of a square-wave input in open-loop (1st link).

friction identification – 1st link

Figure 19: Static friction map for the first link.
Figure 20: Static friction map for the second link.

Figure 21: Static friction map for the telescope.
5 Parametrisation of the first link model

5.1 Crane mechanics

The geometrical parameters were extracted from the CAD drawing of the crane provided by Cranab AB. The dimensions are $a_1 = 157\, \text{mm}$, $a_2 = 262\, \text{mm}$, $b_1 = 181\, \text{mm}$, $b_2 = 1400\, \text{mm}$, $b_3 = 303\, \text{mm}$, $b_4 = 140\, \text{mm}$, $b_5 = 368\, \text{mm}$, $b_6 = 100\, \text{mm}$ and $b_7 = 4\, \text{mm}$. The right-angled triangle with $b_8 = 1410\, \text{mm}$ as hypothenuse and $b_2$ as cathetus gives the angle

$$\alpha_1 = \arccos \left( \frac{b_2}{b_8} \right)$$

in respect of the horizontal axis of the first link. Hence, the given parameters can be transferred to the used coordinate system as introduced in chapter 3. It follows

\[
\begin{align*}
  d_1 &= a_1 = 0.1570\, \text{m} \\
  d_2 &= b_2 \sin (\alpha_1) = 0.1664\, \text{m} \\
  d_3 &= b_5 \cos (\alpha_1) - (b_2 + b_1) \sin (\alpha_1) = 0.1774\, \text{m} \\
  d_4 &= b_3 \sin (\alpha_1) + b_6 \cos (\alpha_1) = 0.1353\, \text{m} \\
  d_5 &= b_4 \sin (\alpha_1) + b_7 \cos (\alpha_1) = 0.0206\, \text{m} \\
  r_1 &= a_2 = 0.2620\, \text{m} \\
  r_3 &= b_2 \cos (\alpha_1) = 1.3901\, \text{m} \\
  r_2 &= b_5 \sin (\alpha_1) + (b_2 + b_1) \cos (\alpha_1) - r_3 = 0.2235\, \text{m} \\
  r_4 &= b_3 \cos (\alpha_1) - b_6 \sin (\alpha_1) = 0.2890\, \text{m} \\
  r_5 &= b_4 \cos (\alpha_1) - b_7 \sin (\alpha_1) = 0.1385\, \text{m}.
\end{align*}
\]
For the equations of motion of the first link the parameters are:

\[ m_t = 650 \text{ kg} \]
\[ d_m = 1.1011 \text{ m} \]
\[ J = 788.0738 \text{ kg m}^2. \]  

(30)

5.2 Crane hydraulics

A good parametrisation requires the values taken from data sheets of the components involved in the system, such as frequency responses, valve opening, cylinder coefficients, etc. These data sheets unfortunately are not provided by manufacturers, and in some cases when they are given they do not give enough information.

In order to estimate values for the unknown hydraulic parameters a number of open loop tests were performed.

The parameters used for the spool dynamics are:

\[ \omega = 2 \pi \cdot 161.2 \]
\[ \xi = 0.481 \]
\[ \Delta t = 7.625 \cdot 10^{-4} \]
\[ L_v = 124.9 \]
\[ L_a = 81.7 \cdot 10^3. \]  

(31)

The bulk modulus has the physical dimensions of pressure. Usually it has the value of \( \beta = 7 \cdot 10^8 \text{ Pa} = 7000 \text{ bar} \), but the value can change by a factor of 10 [3]. The values for the cylinder equations where taken also from CAD drawings to form the geometrical parameters of the chambers. The values used for the model are:

\[ B = 1000 \cdot 10^6 \text{ Pa} \]
\[ g = 9.81 \text{ m/s}^2 \]
\[ A_A = \frac{\pi}{4} \cdot 0.1^2 \text{ m}^2 \]
\[ V_{A0} = 0.02 \cdot \frac{\pi}{4} \cdot 0.02^2 \text{ m}^3 \]
\[ A_B = \frac{\pi}{4} \cdot (0.1 - 0.063)^2 \text{ m}^2 \]
\[ V_{B0} = V_{A0}. \]  

(32)

5.3 Friction model

As already explained an algorithm to estimate friction was applied, during the open loop experimentation. The result of this experimentation is a friction map. This friction map was modelled in Simulink applying look-up tables which use the velocity as input and give a certain amount of current as output that represents the equivalent friction in the system.

The method used, to join the ideal model of the first link with the friction model, was translating the coordinates of the friction model to the starting points \( (v_0, i_0) \).

Various other methods were tried such as polynomials and curve fitting, but the problem with these methods were that after a certain point in the coordinate of velocity they tend to have a sudden fall and rise to very low and high values in the friction value.

5.4 Overview of the model parameters

For the hydraulic model the number of components known and unknown are shown in table 1 and 2, as it can also be seen some values have been approximated or estimated, due to a lag of information, according to experimentation. The valve with which our set up is working is the L90LS and the cylinders are a CRANAB AB production.
Table 1: Number of components for the valve model

<table>
<thead>
<tr>
<th>Known</th>
<th>Approximately Known</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 2: Number of components for the cylinder model

<table>
<thead>
<tr>
<th>Known</th>
<th>Approximately Known</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

In the other hand more information was provided for the geometrical and mechanical part of the crane. Some useful CAD drawings of the set up gave us the appropriate dimensions needed in the model. Something that has not been done is to model the mechanical part in a mechanical program to give us values for the inertia of the first link, though it does not mean that the value calculated is not reliable and besides it does not affect the behaviour of the model according to the real set up. The number of parameters can be seen in table 3.

Table 3: Number of components for the mechanical model

<table>
<thead>
<tr>
<th>Known</th>
<th>Approximately Known</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

In resume, the model works with a total number of parameters of 31 including hydraulic and mechanical part. The number of equations needed to model the first link is the 51, in which we have tried to simplify the number of equations and complexity to make it easy to model, formulate and analyse.
6 Preliminary steps in control design

6.1 Friction compensation

From the control perspective, friction may cause the following effects:

- Tracking errors (steady state errors)
- Limit cycles
- Lack of performance.

Therefore, it is needed to understand frictional phenomena in order to reduce these undesired effects.

A reasonable approach to compensate for friction is to add a feedforward term to the actuated control signal \( u \) using an estimate of the friction force \( \hat{F} \) or friction torque \( \hat{F}_\tau \) respectively. Hence, the equations of motion (23) can be modified as follows:

\[
\begin{align*}
\text{for translation:} & \quad m\ddot{x} = u - F + \hat{F} \approx u \\
\text{for rotation:} & \quad J\ddot{\theta} = u - F_\tau + \hat{F}_\tau \approx u 
\end{align*}
\]

(33)

If the compensation works quite accurate, the motion will only depend on the control input and thus no disturbance due to frictional effects will occur. In our case the actuated control input \( u \) is partly known through the given current \( i \) to the control valve. The friction compensation scheme can be achieved anyway using the pseudo-friction \( i_f \) instead (see section 4.2.2):

\[
\begin{align*}
\text{for translation:} & \quad m\ddot{x} = (i - i_f + \hat{i}_f)k \approx ik \\
\text{for rotation:} & \quad J\ddot{\theta} = (i - i_f + \hat{i}_f)k_\tau \approx ik_\tau
\end{align*}
\]

(34)

As already mentioned, the unknown factor \( k \) is not really important for control purposes. Furthermore, the dynamics from the given control current to the finally effected motion is hidden in this equation. An estimate \( \hat{i}_f \) of the current pseudo-friction \( i_f \) can be obtained using the static friction map that has been already derived. In Figure 23 the compensation scheme is illustrated as block diagram exemplified for the first link. It is very convenient to use the pseudo-friction \( i_f \) instead of the real friction force or torque respectively, because here the compensation can be done by simple addition to the control signal. Note that in practice the added friction estimate should always under compensate the real friction to avoid instabilities. Therefore, a friction gain of \( f_g = 0.95 \) has been introduced as factor for the static map.

Due to the fact that only the position is measured, the velocity has to be estimated as well. This can be done by simple high-pass filtering with a cutting frequency of 10 rad/s. Furthermore, a second order low-pass filter (Butterworth) is connected in series cutting the signal for higher frequencies such as introduced by noise. For the normal working range of \([0; 3]\) rad/s, good estimates can be obtained in magnitude and phase. Unfortunately, the sensor data are quite unreliable for low velocities around zero. This may cause drastic problems especially for the switching process between positive and negative velocity. Therefore, a more elaborate strategy than just filtering has to be applied to obtain a reliable velocity estimation (see Figure 24).

An effective friction compensation can be only guaranteed if the velocity is provided in a reasonable accuracy with a small time delay. In our case there is a generally huge time delay of circa 0.7 seconds between an input to the control valve and the final reaction of the crane actuators and thus the sensor data. Applying Euler’s formula (here exemplified for the angular acceleration \( \ddot{\theta} \))

\[
\ddot{\theta}(t) \approx \frac{\dot{\theta}(t) - \dot{\theta}(t - \Delta)}{\Delta}
\]

(35)
should delete this delay $\Delta$ from the filtered (or ideally measured) angular velocity $\dot{\theta}(t-\Delta)$. If friction compensation is working, the acceleration $\ddot{\theta}$ will be proportional to the given control input $i$ (see equation (34)). Hence, the current velocity $\dot{\theta}(t)$ can be obtained without delay as follows:

$$
\dot{\theta}(t) \approx i(t) \gamma(t) \Delta + \dot{\theta}(t-\Delta).
$$

(36)

Here, $\gamma(t)$ shows up as unknown factor of proportionality; but it is practically chosen as a constant with $\gamma(t) = 0.1$ (dimension disregarded). In the case of the first and second link, the generated control signal is inverted due to the fact that a negative input current results in a positive direction of motion.

Moreover, it is needed to indicate the direction of the motion when the velocity is zero, i.e. the motion has to be initiated. Therefore a negligible constant $\mu$ is introduced that always results in an output of the velocity estimation block in order to pick up an initial friction value out of the static map as far as there is a control signal. Otherwise the system would keep on sticking if no velocity is provided.

As last branch in the velocity estimator, the reference signal $r$ is going to be exploited. This is a convenient step for very small velocities in the range of $[-0.1; 0.1]$ rad/s, where the sensor signal is not reliable. It is assumed that the controller works quite accurate in order to achieve good performance in positioning tasks. Presumably, the real motion is very close to the reference signal then; so the known reference value can be taken instead of the
uncertain one. Here, the velocity is also obtained by a simple high-pass filter. To guarantee a smooth transition between the two signals, a cross-fading is done by using an appropriate bell-shaped curve that gives a fading factor $\alpha(\dot{r})$ (see Figure 25).

![Bell-shaped curve with fading factor $\alpha(\dot{r})$.](image)

Finally, all the mentioned facts are put in an estimation function, where the estimated angular velocity $\hat{\theta}$ is calculated as follows:

$$\hat{\theta} = (1 - \alpha(\dot{r}))\dot{\theta} + \dot{r}\alpha(\dot{r}) + \mu\text{sign}(i).$$ (37)

In the case of translational motion (e.g. for the telescope), this function can be also applied equivalently.

### 6.2 Identification of a nominal plant model

An appropriate control design is based on a sufficient knowledge of the system. In chapter 3, the modelling process for the first link was already shown from the theoretical point of view. There, a quite deep insight into the behaviour and functionality is provided.

Another possibility to derive a nominal plant model can be realised by identification. Here, a sinusoid input is applied to a stable system in order to obtain its frequency response. Afterwards the transfer function can be derived by evaluating the magnitude and phase. Actually, this method is intended to deal with linear closed-loop systems and thus there is a problem if huge non-linear effects are introduced by friction or other uncertainties. In Figure 26 the general plant model of the crane is depicted. The non-linearity is covered by the friction block. If the objective is to identify the nominal plant model, the friction has to be cancelled. This can be done by applying the compensation scheme that has been introduced in the previous section.

As already mentioned, the system has to be stable in order to obtain a reasonable frequency response. This cannot be assumed for the open-loop system of the crane. It rather shows the behaviour of an integrator. Therefore, the identification has to be done in closed-loop mode where a controller guarantees internal stability at least for a certain range of frequencies. The nominal plant model $G_0(s)$ can be computed backwards out of the identified nominal complementary sensitivity function (closed-loop transfer function)

$$T_0(s) = \frac{G_0(s)C(s)}{1 + G_0(s)C(s)}.$$ (38)

If the transfer function of the controller $C(s)$ is known, it follows

$$G_0(s) = \frac{T_0(s)}{C(s)(1 - T_0(s))}.$$ (39)
Normally, a controller is designed according to the properties of the open-loop system regarding internal stability and desired dynamics of the step response. In our case, an empirical tuned PID controller

\[ C(s) = K_p + K_i \frac{1}{s} + K_d \frac{s}{\tau s + 1} \]  

with \( K_p = 2 \), \( K_i = 0.0005 \), \( K_d = 0.001 \) and \( \tau = 1/10 \) has been used in order to obtain a nominal plant model finally. The chosen parameters guarantee internal stability and smooth motions for low frequencies up to 1 rad/s. This controller works in the same quality for the first link, the second link and the telescope. No other parameter set was found to enlarge the bandwidth.

The identification tests were taken for the three closed-loop systems of the mentioned crane actuators. The resulting frequency responses for the first and second link as well as the telescope are quite close to each other. This might be due to the fact that the achieved bandwidth is too small to reveal particular properties. Roughly, a third order system with a DC-gain equal to one can be approximated (see Figure 27), i.e. the nominal complementary
sensitivity function becomes:

\[ T_0(s) = \frac{1}{(s + 1)^3}. \]  

(41)

Moreover, a small delay of circa 0.2 s could have been observed. It is neglected due to the fact that its caused phase lag has almost no influence in the considered bandwidth (dashed line in the phase portrait, Figure 27). An identification for higher frequencies than 1 rad/s has not been achieved yet because the closed-loop system gets unstable in that unknown area. Probably this effect is caused by a resonance behaviour.

Finally, a nominal plant model can be computed applying equations (39) to (41):

\[ G_0(s) = \frac{0.5}{s(s^2 + 3s + 3)}. \]  

(42)

The corresponding Bode plot is depicted in Figure 28. Here, the dominant integral behaviour can be seen.

![Bode Diagram](image)

Figure 28: Bode plot of the computed nominal plant model.

To compute the nominal plant transfer function, small simplifications have been done. The pole \((s + 0.00025)\) is shifted to zero and the integrator constant of 0.49751 is rounded to 0.5. This can be mapped to the modelling error. In the considered bandwidth, i.e. for low frequencies, the modelling error can be neglected, but it becomes increasingly significant for higher frequencies.

However, the identified plant model is not truly representative for the real crane behaviour, it is rather a rough approximation within a limited bandwidth. Further tests with varying controllers might validate the obtained transfer function. Nevertheless, the main objective is still the identification of the frequency response for the true bandwidth bounded by the performance of the actuators. Controllers have to be found that work at least partly for higher frequencies than 1 rad/s.
6.3 Outlook on further steps in development

Commonly, feedforward terms may help to improve dynamic tracking and to minimize the steady-state control error. Therefore, the reference or disturbance signals have to be known or measurable respectively. In our case especially a reference feedforward seems to be applicable because of the permanent availability of the signal. A two-degree-of-freedom architecture offers the possibility to design of a prefilter $H(s)$ for the reference signal [6]. Here, the transient performance of an existing closed-loop system can be affected additionally to the designed controller, even when the reference signal exceeds the allowed closed-loop bandwidth. The resulting transfer function $T_0(s)H(s)$ has to be chosen according to the required frequencies. Of course the bandwidth of the reference response will be limited to the available performance of the actuators.

If repetitive tasks have to be executed, the closed-loop performance can be improved applying Iterative Learning Control (ILC) methods [9]. The accuracy of tracking a given trajectory will rise with each iteration step exploiting the current control error for the next cycle. Disturbances and slightly changing process properties might be compensated as well.

Evidently, the friction estimation can be considered as part of the control system and thus it is valuable to investigate dynamic models in order to improve the friction compensation. As already discussed, dynamic friction models such as the LuGre model do not rely on switching at zero velocity and attempt to cover dynamic effects such as pre-sliding displacement, varying break-away force and frictional lag. Moreover, the parameters of the used friction models could be adjusted on-line by adaptive methods. This is quite useful if properties such as oil temperature or lubrication are changing, but also if the required parameters are only approximately known from the beginning. Assuming a suitable controller that provides internal stability and desired dynamics of the closed-loop system, the tracking error could be used in order to adapt the friction model systematically. For instance, it is possible to adjust the offsets of the static map in that way and the frictional lag could be considered as well by evaluating the velocity. However, this velocity itself is an estimated value due to the fact that only position sensors are used. Even when the developed estimation scheme seems to be quite sophisticated, it might be more reliable to receive the signal directly from either another sensor or an appropriate model connected in parallel to the process.

Nevertheless, the most important step is to design a control system for each actuator of the crane that suits to the demands in tracking and transient performances. Presumably, those controllers will not be much different from each other because of the similar behaviour of the several hydraulic actuators. As already revealed in the previous section, it might end up in a more complex design where multiple controllers are used for particular frequency ranges in order to cover the whole bandwidth of the system. Actually, the fundamental task is still to derive a nominal plant model where the control design will be based on. This can be done by developing a suitable identification procedure in order to investigate the frequency response in the needed bandwidth. The evaluation of a set of candidate model descriptions will result in the most reasonable one. Instead of identification tests, the nominal plant model can be also derived by analysing the equations that have been used in the modelling process. The obtained transfer function or even state-space description will lead to possible design methods like Pole Assignment or LQR. Moreover, a so-called back-stepping procedure allows to start with the control design only for the piston position of the cylinder in order to shape the desired properties for the closed-loop performance. Step by step the controller will be adjusted introducing the other model equations beginning from the cylinder back to the valve and the spool. After appropriate controllers have been found, the focus will be directed on the generation of autonomous motions.
7 Experimental results

7.1 Model versus real world

Due to the limitation in sensors and due to the lack of information of the components in the hydraulic circuits an identification based estimation was not the right answer to solve the problem of parametrisation.

As previously discussed in chapter 5.2 numerous open- and closed-loop experimentations were performed in order to validate and find approximated values for the unknown and not simply measurable parameters.

In the following paragraphs we will show how the model versus the real machine works after finding reasonable approximated values.

7.1.1 Open-loop

For the open loop experimentation a number of different square waves were given to the system and the output was recorded. In this input only the amplitude of the signal was changed for each evaluation. Afterwards this recorded data was used as input in the model, in order to have the same input and compare both results. The figure 29 shows one of the results obtained from this tests, the red line represents the answer of the real crane under the stated conditions and the dotted blue one represents the answer of the model. For this specific experiment shown in the figure the values used were:

\[
\begin{align*}
\text{Amplitude} & = 0.48 \text{ A} \\
\text{Period} & = 10 \text{ s}.
\end{align*}
\]  

(43)

![open-loop experiment for the 1st link](image)

Figure 29: Model versus real crane in open-loop mode for the first link.

7.1.2 Closed-loop

For the closed loop, the designed controller (40) was used. The purpose of the closed loop is to show that the model is able to track a signal in the same way as the real crane with the
designed controller. The closed loop figure can be seen in 30.

\[
\begin{align*}
\text{Amplitude} &= 0.2 \text{ A} \\
\text{Frequency} &= 0.2 \text{ Hz}.
\end{align*}
\] (44)

Figure 30: Model versus real crane in closed-loop mode for the first link. Left graph: model. Right graph: real crane

\section*{7.2 Autonomous motion via motion planner}

\subsection*{7.2.1 Autonomous circle}

With the following experiment an autonomous motion shall be demonstrated. Here, the focus is directed on the quality of the final crane motion in comparison to the desired trajectory. The intention is not to show a sophisticated motion planner, it is simply needed in order to generate appropriate reference values for the crane actuators.

In Figure 31 the deployed motion planner is depicted. Here the boom-tip has to follow a given trajectory. It is quite practical to project a circle onto the \(x\)-\(z\)-plane, because the first and the second link as well as the telescope can be run simultaneously (see Figure 32). One can also evaluate the accuracy of the autonomous crane motion in an easy way. Therefore, the following equations have been used:

\[
\begin{align*}
\psi(t) &= \omega t \\
x^* &= R \cos(\psi(t)) + x_0 \\
z^* &= R \sin(\psi(t)) + z_0 \\
d^* &= D \cos(\psi(t) + \varphi) + d_0.
\end{align*}
\] (45)

The generated function \(\psi(t)\) allows to form a counter-clockwise circle by computing the \(x\) and \(z\) coordinates through the trigonometric functions. The corresponding frequency has been chosen to \(\omega = 0.25 \text{ rad/s}\), i.e. the period for one round is circa 25 s. A radius of \(R = 0.7\) m with its centre at \(x_0 = 3\) m and \(z_0 = -0.25\) m have been reasonably set. Note that the joint of the fist link to the shaft is the origin of the coordinate system. The telescope is not really needed to describe the circle, but it will be used anyway for demonstration purposes. Here, the phase of the cosine function is shifted by \(\varphi = -0.25\) rad. A maximal amplitude of \(D = 0.15\) m and an offset value of \(d_0 = 0.5\) m have been chosen.
If the telescope position $d$ is known, the actual angles of the first link $\theta_2$ and of the second link $\theta_3$ can be calculated from any given position of the boom-tip in the x-z-plane. An unique solution will be derived applying inverse kinematics as follows [14]:

\[
\begin{align*}
\theta_3 &= -\arccos \left( \frac{x^2 + z^2 - l_{12}^2 - (d_0 + d)^2}{2l_{12}(d_0 + d)} \right) \\
\theta_2 &= \pi + \arctan2 \left( x(d_0 + d)\sin(\theta_3) - z \left[ (d_0 + d)\cos(\theta_3) + l_{12} \right] , -z(d_0 + d)\sin(\theta_3) - x \left[ (d_0 + d)\cos(\theta_3) + l_{12} \right] \right)
\end{align*}
\]

where $l_{12}$ represents the length of the first link and $d_0$ the length of the second link. The outcome of this transformation block is given as reference values $\theta_2^*$ and $\theta_3^*$ to the corresponding actuators according to the desired trajectory for the boom-tip (compare Figure 31). Here, the desired telescope position $d^*$ is fed through directly from the motion planner.

In the experiment, the crane was running numerous rounds on the projected circle. The applied PID-controller (see section 6.2, equation (40)) together with the derived friction compensation scheme (see section 6.1) did a quite good job in tracking the given references for each actuator. The experimental result is shown in Figure 33, where the generated trajectory and the final crane motion are super-imposed. Only during the lifting phase there are some wavelike tracking errors around the trajectory. Obviously, the friction estimation is not sufficient in that part. Tests for higher frequencies than 0.25 rad/s have been also performed, but this was accompanied by a significant loss of accuracy.

### 7.2.2 Autonomous semi-ellipsoid with additional gripper actuation

Considering the usual motions of cranes according to their dedicated tasks, semi-ellipsoid trajectories are more reasonable to project as given objective. For instance, things could be
picked up in front of the crane and displaced somewhere else using the gripper additionally. The motion planner that has been derived in the previous section must be adjusted for this purpose. First of all, a ramp sequence $g(t)$, including two pauses of $p = 3 \text{ s}$ at $t = 0$ and $t = p + T/2$, will be generated as follows:

$$
g(t) = \begin{cases} 
0, & \text{if } t \in [0, p] \\
t - p, & \text{if } t \in [p, p + T/2] \\
T/2, & \text{if } t \in [p + T/2, 2p + T/2] \\
t - 2p, & \text{if } t \in [2p + T/2, 2p + T] 
\end{cases}
$$

(47)

where $T = 2\pi/\omega$ represents the time period (see Figure 34). The applied frequency is chosen again to $\omega = 0.25 \text{ rad/s}$. During the pause $p$, the gripper is going to be actuated in open-loop mode. It will simply open and close for demonstration purposes. A sensor is not installed yet. Furthermore, the functions

$$
\psi(t) = g(t)\omega \\
\psi_s(t) = -\frac{\pi}{2}\cos(\psi(t)) + \frac{\pi}{2}
$$

(48)

will be used as arguments for the following trigonometric equations in order to generate a semi-ellipsoid in the $x$-$z$-plane:

$$
x^* = R\cos(\psi_s(t)) + x_0 \\
z^* = \epsilon R\sin(\psi_s(t)) + z_0 \\
d^* = D\cos(\psi(t) + \varphi) + d_0.
$$

(49)
Here, the factor $\epsilon$ determines the dilation in the $z$-dimension. Its value is practically chosen to $\epsilon = 1.3$. In comparison to the previous motion planner, the maximal amplitude of the telescope is changed to $D = 0.3$ m and its phase shift is cancelled by $\varphi = 0$. Furthermore, the centre of the ellipsoid, i.e. the starting position of the semi-ellipsoid, is moved closer to the ground by $z_0 = -0.9$ m. In Figure 35 the described motion is illustrated. The gripper is not shown, but it is connected to the boom tip in reality. The crane is supposed to move among the projected semi-ellipsoid back- and forwards all the time. At each end of the trajectory, the gripper is actuated in open-loop mode during the introduced pauses. It opens and closes softly in order to demonstrate a more practical task for cranes.

The experimental result is comparable to the autonomous circle shown in the previous section. The given references for each actuator are followed quite accurate, but the wave-like tracking errors are especially seen in this kind of motion (see Figure 36). As already mentioned, an insufficient friction estimation for the lifting process might be one reason for this. Probably, there are also some constraints in the valve if all actuators are working at the same time.
8 Conclusions

In this report, modelling issues as well as preliminary steps in the control design of a hydraulic forestry crane were presented. Occurring friction in such heavy-duty machinery may reduce the performance significantly and introduces negative side effects such as tracking errors, large settling times or limit cycles. That is why the main focus was directed on friction modelling and compensation in order to eliminate the mentioned effects.

An appropriate identification procedure for a lumped pseudo-friction model was proposed, where the friction in all the several components of each actuator of the crane is combined in one model expressed as current in respect of velocity. Through experimental tests, characteristic parameters could be obtained and appropriate static friction models were formed consequently for the first link, the second link and the telescope.

A reasonable approach to compensate for friction has been realised by adding a feedforward term to the control signal using an estimate of the current pseudo-friction via the corresponding static friction map. Due to the fact that only the position is measured, the velocity has to be estimated as well. Therefore, an elaborate estimation algorithm has been developed, where especially problems at low velocities around zero are taken into account.

Control design is always based on a sufficient knowledge of the system. An expedient possibility to derive a nominal plant model was taken by experimental identification. Analysing the identified frequency response of the closed-loop system, the transfer function of the plant could be computed backwards. Therefore, an empirical tuned PID controller was used. An identification for higher frequencies than 1 rad/s has not been achieved yet because the closed-loop system gets unstable then. However, the identified plant model is not truly representative for the real crane behaviour, it is rather a rough approximation within a limited bandwidth. The main objective is still to derive a nominal plant model for the true bandwidth, where the control design will be based on. An outlook on further steps of improvements was discussed as well.

In conclusion, the derived control system consists of an empirical tuned PID controller with an additional feedforward term for friction compensation. As experimental result of this
report, two examples of an autonomous motion are presented. The tracking of the projected trajectories is quite promising for the future progress.

The modelling method presented in this report suggests an easy and rapid way of modelling complex systems taking the advantage of the MATLAB/Simulink package. A fast and easy way of changing parameters for the coefficients of the mathematical models enables the examination of effects involved in the system. It opens also the possibility to study the reaction of the system to new type of components such as faster valves, different cylinders, etc., without having to use the real machines and components.

The model encountered, which is based on differential equations of each component in the system, performs the tasks we expected and is comparable to the functioning of the real machine in the first link. The experimental results – comparing real data and simulated data – show a promising step in modelling and this encourages us to continue further developing the model not only for the first link but the whole system.
References


