Motion Planning and Dynamical Positioning for a Fleet of Underactuated Ships

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I. INTRODUCTION

This paper is devoted to the problems of motion planning and feedback stabilization for dynamical underactuated ship. We use the models taken from [1], where it is suggested to describe the behavior of a ship by three independent variables: the angle that the ship makes with the direction to the north and and two coordinates that define the position of the ship in the horizontal plane, which are usually taken as the north-east coordinates.

It is important to notice that the dynamics of the ship differs from the dynamics of a rigid body on a plane. This is due to hydrodynamical effects (behavior of ambient water) and due to the presence of friction terms from the motion in the water with both linear and quadratic velocity dependencies.

It is assumed that the ship has only two control inputs that correspond to two generalized forces acting on the ship. They are the propeller thrust and the angle of the rudder. The absence of the 3rd control input makes the ship underactuated.

In this paper we consider a formation of several ships. We assume that the specifications for desired behavior for each ship or a family of ships is given by a geometrical path and an orientation along the path. In general, the specifications can be updated off-line or/and on-line. They are seen as implicit instrumental tools for further controller designs.

Projecting dynamics of the ship (the family of ships) onto such a geometrical path (paths), we obtain a description of all feasible motions of the ship (formation) along such path(s) without its explicit time parametrization. For the dynamical positioning problem, the search of equilibria of the projected dynamics on such path(s) and the analysis of their stability (instability) are of the highest interest. Below we briefly discuss possible solutions with different topologies of a ships’ formation.

II. PROJECTED DYNAMICS FOR ONE UNDERACTUATED SHIP MODEL

We start with presenting a path planning problem for one ship with three degrees of freedom and two control inputs.

Feasible motions of the ship model are computed for an illustrative example taken from [1, Example 10.1, p. 410]. It describes a high speed container ship of length $L = 175$ m and displacement volume $21,222$ m$^3$, which is equipped with one rudder and one forward thrust propeller.

Let

$$\eta = [n, e, \psi]^T$$ (1)

be the North-East position and the yaw angle, respectively, and

$$\nu = [u, v, \tau]^T$$ (2)

be the velocity vector in the body frame. The kinematics equation is

$$\frac{d}{dt}\eta = R(\psi)\nu, \quad \nu = R(\psi)^T \frac{d}{dt}\eta,$$ (3)

where

$$R(\psi) = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

is the rotation matrix in yaw. The surge speed equation and the steering equations (sway and yaw) are assumed to be decoupled. The ship dynamics, written in the body frame, are given by

$$M\dot{\nu} + N(\nu)\nu = B(\nu)\tau + R(\psi)^T w$$ (4)

where $\tau = [T, \delta]^T$ is the control input with $T$ being the propeller thrust and $\delta$ being the angle of the rudder, $w$ is the vector of environmental disturbances, the matrix functions $M, N,$ and $B$ are

$$M = \begin{bmatrix} m - X_u & 0_{1\times2} \\ 0_{2\times1} & I_{2\times2} \end{bmatrix}$$ (5)

$$N(\nu) = \begin{bmatrix} -X_u - |u|X_u|u| & 0_{1\times2} \\ 0_{2\times1} & -\frac{\nu}{L} \begin{bmatrix} a_{11} & L a_{12} \\ a_{21} & a_{22} \end{bmatrix} \end{bmatrix}$$ (6)

$$B(\nu) = \begin{bmatrix} (1 - t_d) & 0 \\ 0 & \frac{1}{L^2} b_{11} \end{bmatrix}$$ (7)
Here \( X_i \), \( a_{ij} \), and \( b_{ij} \) are the hydrodynamic coefficients\(^1\), \( t_d \) is the thrust deduction number, \( t_d \in (0,1) \), and \( U = \sqrt{u^2 + v^2} \) is the total speed.

We assume that the vector \( w \) of environmental disturbances in (4) is constant. Suppose that the control inputs \( T \) and \( \delta \) are chosen to preserve a given path invariant for ship movements. We describe below all the possible motions of the ship.

**Theorem 1:** Suppose that the path and the yaw angle are defined by the \( C^2 \)-smooth functions of the new independent variable \( \theta \):

\[
n = \phi_1(\theta), \quad e = \phi_2(\theta), \quad \psi = \phi_3(\theta), \quad (8)
\]

and that there exists a control input \( \tau^* = [T^*, \delta^*]^T \), which makes the relations (8) invariant for the ship dynamics (4). Then \( \theta \) is one of the solutions of the dynamical system

\[
\alpha(\theta) \ddot{\theta} + \beta_1(\theta) \dot{\theta}^2 + \beta_2(\theta) \dot{\theta} + \gamma(\theta) = 0 \quad (9)
\]

The explicit formulae of the functions \( \alpha(\theta) \), \( \beta_1(\theta) \), \( \beta_2(\theta) \), and \( \gamma(\theta) \) are given in the proof presented in [2].

A test for asymptotic (in-)stability of equilibria of (9) is given in the next statement.

**Theorem 2:** Let \( \theta_0 \) be an equilibrium point of the system (9), that is the point where \( \gamma(\theta_0) = 0 \). Suppose that the functions \( \alpha(\theta) \), \( \beta_1(\theta) \), \( \beta_2(\theta) \), and \( \gamma(\theta) \) are such that the constant

\[
\omega_0 = \left. \frac{d}{dt} \frac{\gamma(\theta)}{\alpha(\theta)} \right|_{\theta = \theta_0} \quad (10)
\]

is positive and that the inequality

\[
\frac{\beta_1(\theta_0) + \beta_2(\theta_0)}{\alpha(\theta_0)} > \frac{\beta_1(\theta_0) - \beta_2(\theta_0)}{\alpha(\theta_0)} \quad (11)
\]

holds. Then, the equilibrium \( \theta_0 \) of (9) is asymptotically stable. Moreover, if the sign of inequality (11) is opposite, then the equilibrium \( \theta_0 \) of (9) is unstable.

Further details and a feedback design method for dynamical positioning of one ship can be found in [2].

**III. DYNAMIC POSITIONING FOR FORMATIONS OF UNDERACTUATED SHIPS**

Equipped with the dynamical positioning algorithms elaborated for one ship, one can readily consider DP-problems for formations of ships with different topologies in communication and control strategies.

One of the solutions for an automatic reallocation and bias correction in a chain of pontoons used for shaping a landing run on sea is illustrated on Fig. 1 and 2. On the plots we show the process of building a formation of 3 ships. The first ship is assigned to remain on a particular geometrical path in the inertia frame. The two other ships form a chain behind the first one. The geometrical constraints for the second and the third ships are chosen as circles with the centers corresponding to the position of center of mass of the ship previous in the chain. In the beginning phase, the formation reaches the stable equilibrium formed by environmental forces \( w_1 \) acting from the west. After some time, during the second phase, the environmental forces abruptly change direction. As a result, the formation reaches a new stable equilibrium after transition.

Fig. 1. The process of building a formation of 3 ships. The formation reaches a stable equilibrium due to environmental forces \( w_1 \) acting from the west.

Fig. 2. During the second phase, the formation reaches another stable equilibrium due to environmental forces \( w_2 \) acting from the south.

\(^1\)The numerical values for the example are \( m = 21.2 \cdot 10^6 \), \( X_0 = -6.38 \cdot 10^5 \), \( a_{11} = -0.7072 \), \( a_{12} = 0.286 \), \( a_{21} = -4.1078 \), \( a_{22} = -2.6619 \), \( b_{11} = -0.2081 \), \( b_{21} = -1.5298 \).

**REFERENCES**
