Rate Distortion Theory
(INFORMATION THEORY)

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INFORMATION THEORY

It is a branch of the mathematical theory of probability that quantifies the concept of information.

1. Information Entropy
2. Data Transmission
3. Rate Distortion Theory
Rate Distortion Theory

RATE:
- It is the number of bits per data sample to be stored or transmitted.

DISTORTION:
- It is defined as the variance of the difference between input and output.
- 1. Hamming distance
- 2. Squared error
Rate = 0.1 bits per pixel  Rate = 1.0 bits per pixel

Example of Rate and Distortion
Examples of Distortion
Variance of Input and Output Image
(Example of distortion)
Rate distortion theory is the branch of information theory addressing the problem of determining the minimal amount of entropy or information that should be communicated over a channel such that the source can be reconstructed at the receiver with given distortion.
Lower the bit rate $R$ by allowing some acceptable distortion $D$ of the signal.
The spacing of discrete values in the range of a signal is called the quantization of that signal. Quantization is usually thought of as the number of bits per sample.

**EXAMPLES:**

1. Black and white images (1 bit per pixel)
2. 24 bit color image
Black and White Image (1 Bit per Pixel)
Example of a 24-bit Color image
The functions that relate the rate and distortion are found as the solution of the following minimization problem:

\[
\min_{Q_Y|X(y|x)} I_Q(Y;X) \text{ subject to } D_Q \leq D^*. 
\]

Mutual information \( I(Y;X) = H(Y) - H(Y | X) \)

\[
D_Q = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_{X,Y}(x,y)(x-y)^2 \, dx \, dy = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Q_{Y|X}(y|x)P_X(x)(x-y)^2 \, dx \, dy
\]
Solving the minimization can be done for few cases, of which the following two are the most wellknown ones.

1. Memoryless (independent) gaussian source.
2. Gaussian source with memory.
Memoryless Gaussian source

If $P_x(X)$ is gaussian, variance is $\sigma^2$. And if we assume that successive samples of the signal $x$ are stochastically independent, we find the following analytical expression for the rate distortion function.

$$R(D) = \begin{cases} 
\frac{1}{2} \log_2(\sigma_x^2/D), & \text{if } D \leq \sigma_x^2 \\
0, & \text{if } D > \sigma_x^2
\end{cases}$$
Rate Distortion function
Thank you